

NON-CLASSICALITY INDICATORS FOR ENTANGLED AND SQUEEZED NUMBER STATES

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Many non-classicality indicators are used to measure quantum effects of different systems. Kenfack's and Sadeghi's non-classicality indicators are introduced in terms of the amount of Wigner function's negativities and interferences in phase space quantum mechanics, respectively. They are, effectively, applied for some real distribution functions. In this paper, we compared these non-classicality indicators for the entangled state of photonic number states in the Wigner, Husimi and Rivier representations. It is shown that for a two-level entangled state, Sadeghi's indicator has more benefits with respect to the Kenfack's indicator. For the two-level entangled state, we show a correspondence between the Sadeghi's non-classicality indicator and the Von Neumann entropy. It is also shown that for the superposition of squeezed number state the Sadeghi's (and no Kenfack's) non-classicality indicators is sensible with respect to the squeezing parameter for a superposition of squeezed number states

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I. INTRODUCTION

The phase space formulation of quantum mechanics, presented by Wigner in 1932 [1], offers a classical like formulation. He proposed Wigner distribution functions (WDFs), which in general, were real and non-positive [2, 3]. Some authors are believed the negativity of Wigner distribution functions has indicated to some nonclassicality phenomena [4, 5]. Kenfack and Życzkowski invent a nonclassicality indicator which measured the negativity of Wigner distribution function [6]. Their nonclassicality indicator has been applied for different quantum systems [7–9]. As well as WDF, there is many distribution functions in different phase space representations e.g.: Kirkwood distribution function (KDF) [10], Husimi distribution function (HDF) [11], Rivier distribution function (RDF) [12] and so on [13–16]. Among them the HDF is positive definite and has no negativity at all [3]. The expectation values of physical quantities are assumed to be independent of different phase space representations, therefore all distribution functions should be, in general, equivalent [2, 3]. Application of a specific phase space representation for special systems, have their benefit in calculation [2, 3]. The Kenfack and Życzkowski's non-classicality indicator is just defined in terms of WDF [6]. In general it doesn't suitable for other real distribution functions [17]. It specially vanishes for the positive definite distribution functions like HDFs. The equivalence of distribution functions lead authors to ask, "if the negativity of WDFs indicate to the physical quantum effects,

what about them in other distribution functions?"

Clearly the Kenfack and Życzkowski's nonclassicality indicator (which doesn't vanish for WDFs) vanishes for HDF. To remove this inconsistency another nonclassicality indicator is introduced by Sadeghi *et al.* [17] based on the interference of quantum states. It is shown that the Sadeghi's nonclassicality indicator works properly for some real distribution functions, e.g. Husimi, and Rivier as well as Wigner [17]. In this paper these nonclassicality indicators are applied for the entangled and a superposition state of squeezed number states to measure the amount of entanglement and squeezing parameter. As an example; the entanglement of a two-level and squeezed number states are investigated to compare the benefits of two indicators. In the next section we have a brief review about the Kenfack and Życzkowski, and Sadeghi's nonclassicality indicators. Section 3 and 4 belong to the calculation and comparison of nonclassicality indicators for the entangled photonic and squeezed number states, respectively. Section 5 is devoted to the conclusions.

II. NONCLASSICALITY INDICATORS

The Wigner distribution function for the state $|\psi\rangle$ is defined by [1]

$$W(q, p) = \int_{-\infty}^{\infty} dx \langle q + \frac{x}{2} | \psi \rangle \langle \psi | q - \frac{x}{2} \rangle, \quad (1)$$

Kenfack and Życzkowski define a nonclassicality indicator which depends on the amount of negativity in Wigner distribution function as follows[6]:

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$$\begin{aligned}\delta_w &= \int_{-\infty}^{\infty} [|W(q,p)| - W(q,p)] dq dp \\ &= \int_{-\infty}^{\infty} |W(q,p)| dq dp - 1,\end{aligned}\quad (2)$$

The nonclassicality indicator δ_w is just defined in terms of Wigner distribution function and is equal to zero for coherent and squeezed vacuum states, for which the corresponding Wigner distribution functions are non-negative. It is also not sensitive to the squeezing parameter, because the negativity of Wigner distribution function is conserved due to squeezing operation. This nonclassicality indicator doesn't work for other distribution functions, like Husimi or Rivier, properly [17]. Another nonclassicality indicator is introduced by Sadeghi *et al.* according to the interference of quantum states in phase space [17]. This nonclassicality indicator is defined by

$$\eta = \frac{\sum_{ij} \int_{-\infty}^{\infty} [|f_{ij}| - f_{ij}] dq dp}{\sum_{ij} \int_{-\infty}^{\infty} [|f_{ij}| + f_{ij}] dq dp}.\quad (3)$$

where the phase space distribution function for a superposition of quantum states $\psi = \psi_1 + \psi_2$, is divided into four parts $f = f_{11}(\psi_1) + f_{22}(\psi_2) + f_{12}(\psi_1, \psi_2^*) + f_{21}(\psi_2, \psi_1^*)$ (the last two parts are interference terms). In Eq. (3), f_{ij} 's are different parts of distribution function and sum is over all parts (interference and non-interference terms). This nonclassicality indicator is bounded between 0 and 1. This indicator is also applied for some different real distribution functions, e.g. Wigner, Husimi and Rivier. It is shown that the behavior of the nonclassicality indicator η doesn't depend on the phase space representations for the Schrödinger cat and thermal states [17].

III. NONCLASSICALITY INDICATORS FOR ENTANGLED STATES

The entangled state between the ground and first excited number states (or the first excited and second excited number states, and so on) is given by $|\psi\rangle = a|0,1\rangle + (1-a^2)^{1/2}|1,0\rangle$ (or $|\psi\rangle = a|1,2\rangle + (1-a^2)^{1/2}|2,1\rangle$, etc.). The parameter " a " is supposed to be a real number for simplicity. The corresponding Wigner function is given by

$$W(q,p) = W_{11} + W_{22} + W_{12} + W_{21},\quad (4)$$

where

$$W_{11}(q,p) = \frac{2a^2}{\pi^2} (q_1^2 + p_1^2 - \frac{1}{2}) \exp[-q_1^2 - p_1^2 - q_2^2 - p_2^2],\quad (5)$$

$$\begin{aligned}W_{22}(q,p) &= \frac{2(1-a^2)}{\pi^2} (q_2^2 + p_2^2 - \frac{1}{2}) \\ &\exp[-q_1^2 - p_1^2 - q_2^2 - p_2^2],\end{aligned}\quad (6)$$

$$W_{12}(q,p) + W_{21}(q,p) = \frac{4a\sqrt{1-a^2}}{\pi^2} (q_1q_2 + p_1p_2) \exp[-q_1^2 - p_1^2 - q_2^2 - p_2^2],\quad (7)$$

where q_1, q_2, p_1, p_2 are the generalized coordinates and momenta in phase space and the interference terms $W_{12}(q,p) = W_{21}(q,p)$. Clearly the nonclassicality indicator has a constant values $\delta_w = 0.426$ for entangled between ground and first excited states (and $\delta_w = 0.653$ for the first and second excited states), independent on the value of " a ". It is also a constant value for Rivier distribution function and vanishes for the Husimi distribution function. Therefore this nonclassicality indicator is not suitable to indicate the entanglement property of this system and has not a correspondence with the Von Neumann entropy for this system which is shown in Fig. 1.

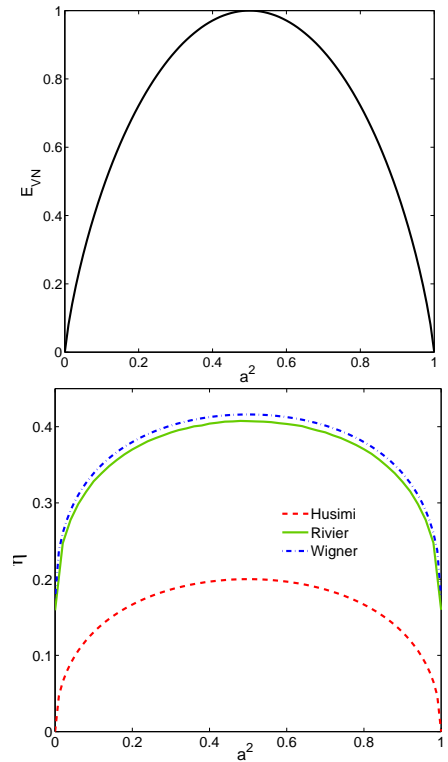


FIG. 1: (Color online) Left) The Von Neumann entropy, E_{VN} , for an entanglement state of first and second excited state of harmonic oscillator vs. a^2 . The parameter a is a superposition state constructor and is supposed to be a real number for simplicity. Right) The non-classicality indicator η for an entanglement state of first and second excited state of harmonic oscillator vs. a^2 for Wigner, Husimi and Rivier distribution function.

Now we investigate the behavior of nonclassicality indicator η for the same state in the Husimi, Rivier, and Wigner representation. The results are plotted in Fig. 2. The nonclassicality indicator η has a constant value in the end points $a = 0, 1$ [18]. The nonclassicality indicator η

has also a maximum at $a = \pm 1/2$ which is corresponding to the maximum entanglement of the Bell states. This maximum point is clearly independent of phase space representations, e.g. Wigner, Rivier and Husimi, which are investigated in this paper. To show this correspondence more clearly, it is suitable to compare the nonclassicality indicator η in different representations which is plotted in Fig. 2 and the Von Neumann entropy in Fig. 1. Although the nonclassicality indicator has different values for different distribution functions, its behavior has a suitable correspondence with the Von Neumann entropy. Application of this method for the entangled state of first and second photonic excited state, and other entangled states, is straight forward. There is a correspondence between the Kenfack and Życzkowski nonclassicality indicator δ , the Sadeghi's nonclassicality indicator η and the Von Neumann entropy for these entangled states.

IV. NON-CLASSICALITY INDICATORS FOR THE SQUEEZED STATES

The squeezed number states are another example from nonclassical states. The Kenfack and Życzkowski nonclassicality indicator δ is not sensitive to the squeezing parameter, due to the amount of negativity for the squeezed state is conserved in squeezing operation. In this section we consider two superposition states:

- 1) The superposition of ground and squeezed ground number states $|\psi_{00r}\rangle = (1 - a^2)|0\rangle + a|0, r\rangle$,
- 2) The superposition of ground and squeezed first excited states $|\psi_{01r}\rangle = (1 - a^2)|0\rangle + a|1, r\rangle$, where "a" and "r" are real probability amplitude and squeezing parameter, respectively. Four parts of Wigner function for $|\psi_{00r}\rangle$ are given by

$$W_{11}(q, p) = \frac{1 - a^2}{\pi} \exp[-q^2 - p^2], \quad (8)$$

$$W_{22}(q, p) = \frac{a^2}{\pi} \exp[-e^{2r} q^2 - e^{-2r} p^2], \quad (9)$$

and

$$W_{12}(q, p) + W_{21}(q, p) = \frac{2a\sqrt{2 * (1 - a^2)}}{\pi\sqrt{1 + e^{2r}}} \cos\left(\frac{2qp(e^{2r} - 1)}{1 + e^{2r}}\right) \times \exp\left[\frac{r}{2} - \frac{2e^{2r}q^2}{1 + e^{2r}} - \frac{p^2}{1 + e^{2r}}\right]. \quad (10)$$

Clearly, the nonclassicality indicator η for $|\psi_{00r}\rangle$ state is plotted in Fig. 3A. The nonclassicality indicator η is an increasing function vs. the squeezing parameter r . The similar result is obtained for $|\psi_{01r}\rangle$ which is plotted in Fig. 3B. Therefore the squeezing parameter is also a measurable quantity with Sadeghi's non-classicality indicator.

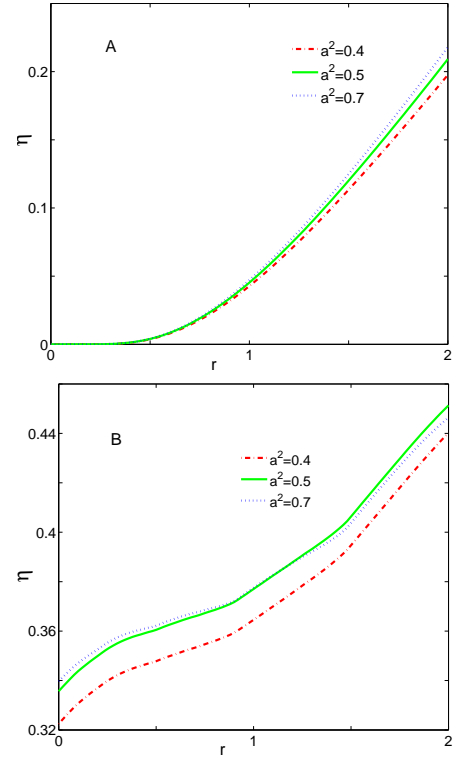


FIG. 2: (Color online) The non-classicality indicator η for squeezed states, A: the superposition of ground and squeezed ground state of number state $|\psi_{00r}\rangle$; B: the superposition of ground and squeezed first excited state $|\psi_{01r}\rangle$ vs. r for different values of probability amplitude in the Wigner representation.

V. CONCLUSION

The nonclassicality indicators δ is applied just for Wigner distribution function in phase space but the nonclassicality indicator η is applicable for more real distribution functions like; Husimi and Rivier. This nonclassicality indicator is applied to indicate to the entanglement which is one of the most important nonclassical phenomena. For an example, the entangled states of two eigenstates are used. For an entangled state consist of the ground and the first excited state the nonclassicality indicators δ has a constant value and therefore is not a suitable indicator for entanglement. In the other hand, the nonclassicality indicator η has a maximum for a Bell state and has a correspondence with the Von Neumann entropy, for the investigated system. Developing to the other entangled states, e.g. the first and second excited states, is straight forward. For these systems the nonclassicality indicator δ has also a correspondence to nonclassicality indicator η and Von Neumann entropy, and has a maximum according to the Bell state which is independent of phase space representations. Also, the nonclassicality indicator η has more correspondence with the squeezing parameter r .

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